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THE HETERODYNING LOCK-IN AMPLIFIER

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1. INTRODUCTION

This note describes a new type of lock-in amplifier introduced by ITHACO in January 1972 as the DYNATRAC® 391 and in particular, the slightly modified and improved version of the instrument (February 1973) known as the DYNATRAC 391A (see figure 1).

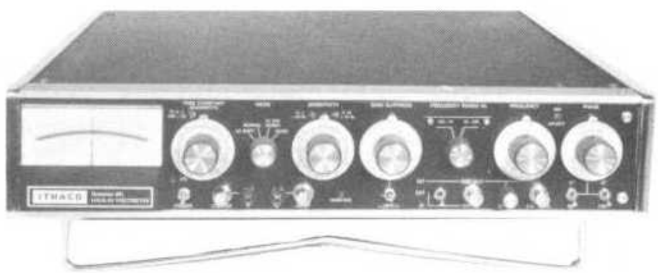


FIGURE 1 DYNATRAC 391A

In the past, the prospective purchaser of a lock-in amlifier had to choose between two types of instrument which essentially differed only in the design of the front-end (i.e. signal input) a.c. amplifier used. These two types of input amplifier can be summarized as follows:

a) BROADBAND A broadband front-end with its flat frequency and phase response can handle an input signal whose frequency drifts or changes, without producing a corresponding phase drift or change between its input and output signals. A broadband front-end cannot handle situations where the signal to be measured is accompanied by extremely large noise or interfering signals, since it overloads under such conditions. In addition, a broadband instrument has harmonic responses. That is the instrument will respond to input signals which are at odd harmonics of the reference frequency (see appendix D).

b) NARROW–BAND A (properly designed) tuned or high Q front-end will attenuate harmonic responses and can reduce most interfering or noisy signals to a safe level and avoid overloading. It does so however, at the expense of phase stability. Even a small change in the frequency of a signal passing through a high Q filter will cause a significant change in the signal phase and hence an unacceptable measurement error. The same error can result with a constant frequency signal if the center-frequency of the filter varies and such frequency instability is common with conventional high Q filters. Such filtering normally has no effect whatever on the signal/noise improvement effected by the instrument (see appendix G).

The design of the DYNATRAC 391A is significantly different from conventional instruments in that heterodyning is used in the front-end of the instrument to eliminate harmonic responses and provide a front-end bandpass filter which automatically tracks changes in the frequency of the signal to be measured. With such a frequency varying signal, this tracking filter action eliminates the manual retuning required with conventional lock-ins using front-end filtering and does not have the phase-instability problems associated with such instruments. The DYNATRAC 391A is therefore as simple to use as a wide-band lock-in but does not have the susceptibility to overloading which is normally a characteristic of instruments with no effective front-end filtering.

The novel design of the DYNATRAC 391A provides exceptional performance in such critical areas as overload capability, output stability, sensitivity and self-noise without a correspondingly high cost. Other significant features that will be described, include the floating input which effectively eliminates the ground loop problems often encountered in the measurement of low-level signals, and the provision of a noise mode (option) which allows the DYNATRAC 391A to be used as a variable frequency, variable bandwidth noisemeter.

As noted above, the design of the DYNATRAC 391A was prompted by the shortcomings of conventional lock-ins. These deficiencies, together with a considerable amount of background information are discussed in detail in the appendices to this note.

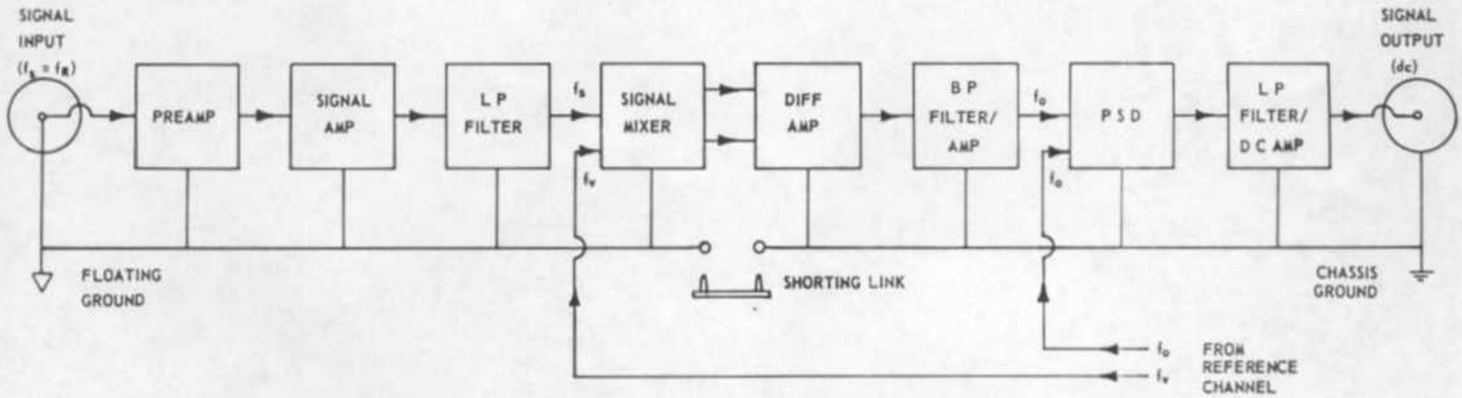


FIGURE 2 DYNATRAC 391A SIGNAL CHANNEL

2. SIGNAL CHANNEL

A simplified block diagram of the signal channel of the DYNATRAC 391A is shown in figure 2. The input signal is amplified by the preamplifier and the variable gain signal amplifier before the signal mixer. The very low self-noise of the instrument is determined by the preamplifier and figure 3 shows the noise figure contours for the instrument. A 24dB/octave low-pass filter (LPF) inserted between the amplifier and the mixer, attenuates noise and unwanted frequencies above the frequency range in use.

The signal mixer outputs are differential and are fed to the differential amplifier. The preamp, signal amp, LPF and signal mixer are all supplied from a separate floating power supply. Any voltages on the input floating ground caused by ground loops will appear as common mode voltages on the differential amplifier inputs and are therefore effectively eliminated. The DYNATRAC 391A may be

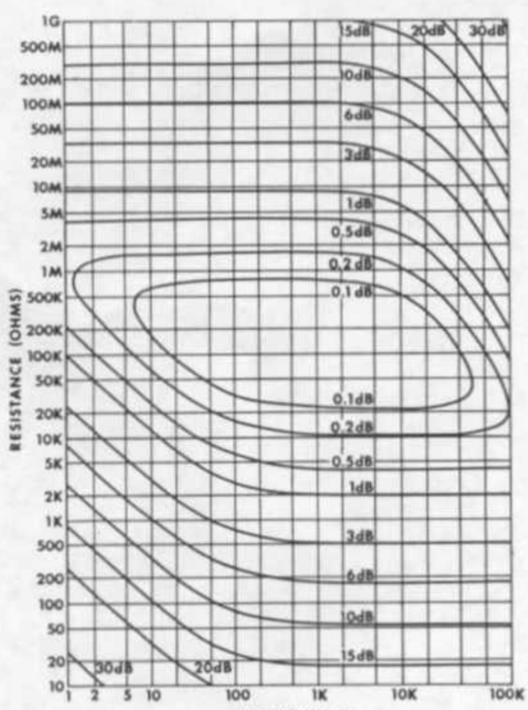


FIGURE 3 TYPICAL NOISE FIGURE CONTOURS

operated in a differential input mode providing that the input floating ground is connected to a low impedance source (< 100Ω). Alternatively, for bridge balancing and other high source impedance applications, a Model 168 differential preamplifier may be used.

If the experiment source has no ground connection e.g. battery powered, the floating and chassis grounds of the DYNATRAC 391A should be connected together with the shorting link provided with the instrument. For normal operation, the link should be removed.

The first mixing of the signal occurs in the signal mixer. The mixing frequency is

$f_v = f_o + f_r$ where

f_v = frequency of a constant amplitude, square-wave signal provided by a variable frequency oscillator (VCO) in the reference channel

f_o = frequency of constant amplitude sine-wave and square-wave signals provided by a fixed frequency oscillator in the reference channel

f_r = frequency of the reference or synchronizing input to the instrument

If the signal has the frequency f_s, the mixer output consists of difference and sum frequencies

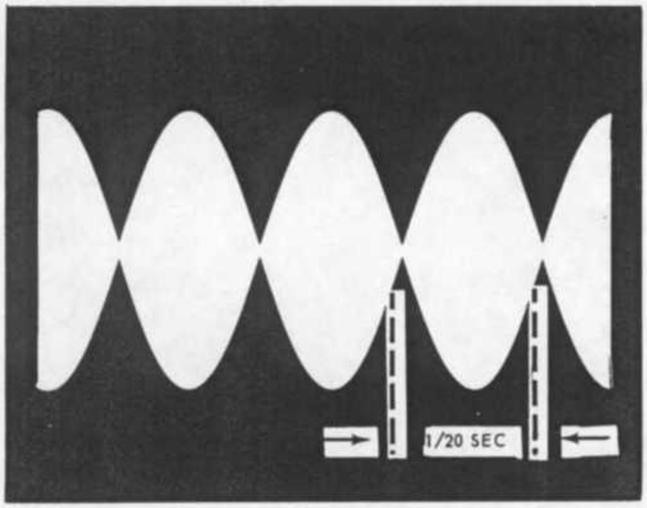
$f_v - f_s$ and $f_v + f_s$

i.e. $f_o + f_r - f_s$ and $f_o + f_r + f_s$

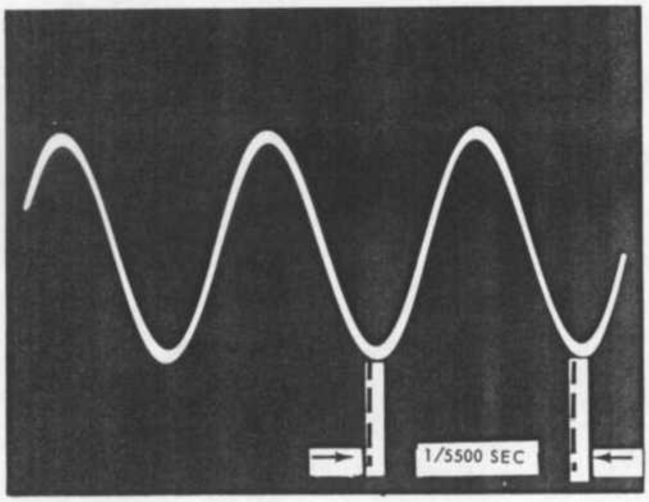
Since wanted signals have the same frequency as the reference ($f_s = f_r$) the mixer output contains the frequencies:

f_o and $f_o + 2f_r$

The first of the two mixer products is equal to the constant intermediate (IF) frequency f_o and is passed unattenuated by the narrow bandpass filter/amplifier before being detected in the phase-sensitive detector (PSD). The other mixer product $f_o + 2f_r$ will be attenuated somewhat



a) f_R = 10Hz



b) f_R = 1KHz

FIGURE 4 PSD INPUT SIGNALS FOR f_o = 5.5kHz

(depending on f_r) by the bandpass filter/amplifier. Figure 4 shows the output of the bandpass filter/amplifier for an f_o frequency of 5.5kHz. In figure 4a, f_r = 10Hz so that $f_v = f_o + f_r$ = 5.51kHz and the unwanted sum frequency $f_o + 2f_r$ = 5.52kHz, is passed by the bandpass filter/amplifier, resulting in the 'two-tone' signal shown. In figure 4b, f_r = 1kHz so that f_v = 6.5kHz and the unwanted sum frequency $f_o + 2f_r$ = 7.5kHz, is severely attenuated by the bandpass filter and results in an almost sinusoidal input to the PSD.

The $2f_r$ component shown in figure 4a is of no consequence since it cannot cause overloading and merely results in a $2f_r$ ripple superimposed on the dc output of the PSD. This $2f_r$ ripple is also present on the output of a conventional lock-in amplifier and in practice is removed by proper selection of output time constant.

Input signals of frequency

$$f_s = 2f_o + f_r$$

will also be converted to f_o by the signal mixer and be detected by the PSD. This spurious response is usually called the image frequency. All possible image frequencies are well above the cut-off frequency of the LPF and are therefore sufficiently attenuated to be of no significance. A slight imbalance of the mixer will let input signals of frequency f_o through, but any f_o signals at the input will also be effectively attenuated by the LP filter.

An important difference between the DYNATRAC 391A and previous lock-in amplifier designs, is in its operating frequency range. The DYNATRAC 391A has an overall operating frequency range of 0.1Hz to 200kHz split into five overlapping frequency ranges. The design philosophy here is as follows:

COLOR CODE	NOMINAL FREQUENCY RANGE (Hz)	USABLE FREQ. RANGE EXT MODE (Hz)	USABLE FREQ. RANGE INT MODE (Hz)	LP CUTOFF FREQ (Hz)	IF FREQ (Hz)	APPROX. IF BW (Hz)	MAX SETTLING TIME	MAX SWEEP RATE Hz/sec	MIN SWEEP TIME f TO 10f (Sec)
BROWN	.1 – 1 1 – 10	.1 – 2 .5 – 20	.1 – 1.1 1 – 11	25	55	±1	15 min 100 sec	7 X 10^{-5} .007	13000 1300
RED	1 – 10 10 – 100	.5 – 20 5 – 200	1 – 11 10 – 110	250	550	±10	100 sec 10 sec	.007 .7	1300 130
ORANGE	10 – 100 100 – 1K	5 – 200 50 – 2K	10 – 110 100 – 1.1K	2.5K	5.5K	±100	10 sec 1 sec	.7 70	130 13
YELLOW	100 – 1K 1K – 10K	50 – 2K 500 – 20K	100 – 1.1K 1K – 11K	25K	55K	±1K	1 sec .1 sec	70 7K	13 1.3
GREEN	10K – 100K	5K – 200K	10K – 110K	250K	480K	±10K	.02 sec	300K	.3

TABLE 1 FREQUENCY RANGES

1. An experimenter does not normally work at 0.1Hz on Monday and 200kHz on Tuesday. Normally a 400:1 frequency range such as 0.5Hz–200Hz, or 50Hz – 20kHz provides a more than adequate frequency range for his experiments.

2. Limiting the frequency range allows each frequency range to be optimized. In the signal channel for example, a change of frequency range automatically optimizes the LP and BP filters (see table 1).

3. A reduction in frequency range allows a significant reduction in costs which is passed on to the buyer.

Frequency ranges can be changed quickly and simply by replacing one set of four plug-in printed circuit cards by another set. The plug-in cards require no adjustment when re-inserted in the instrument for which they were calibrated except possibly to re-zero the output dc voltage with the rear panel control provided. (Use HI DYN RANGE mode, 100mV sensitivity and no signal input) Card sets are color coded for easy identification and unused plug-in cards can be conveniently stored in containers provided for that purpose.

Figure 5 shows the frequency response of the bandpass filter used in the DYNATRAC 391A – in this case for the orange or 5Hz–2kHz frequency range card set. For each frequency range, the bandpass filter is tuned to a fixed frequency thus increasing its stability and simplifying operation. The bandpass filter also attenuates most noise and unwanted signals because, referred to the signal input, it acts as a constant bandwidth filter which tracks the reference frequency. The moderate value of Q used in the bandpass filter (approximately 22) provides sufficient selectivity without sacrificing phase stability.

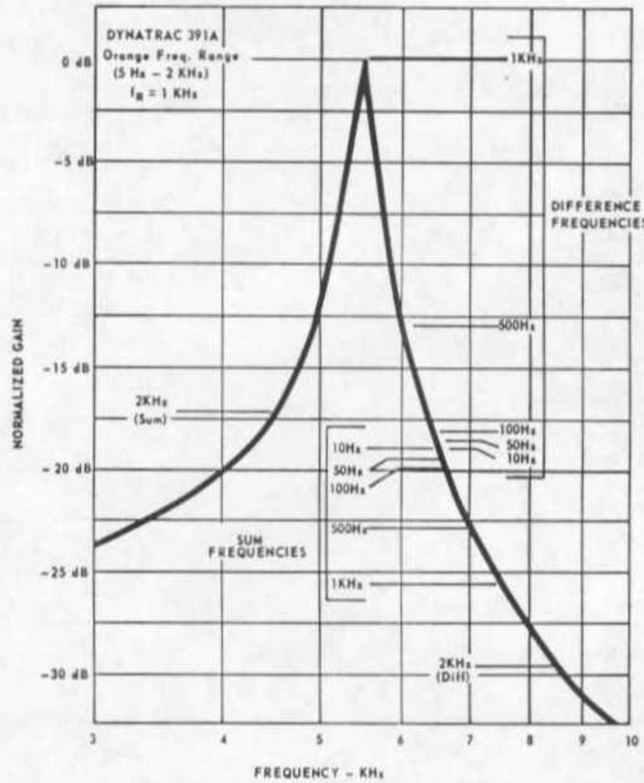


FIGURE 5 BANDPASS FILTER RESPONSE

Figure 5 also shows the effective attenuation of the filter to the sum and difference frequencies caused by asynchronous input signals at frequencies of 10Hz, 50Hz, 100Hz, 500Hz and 2kHz (synchronous input frequency in this case is 1kHz). As with any lock-in amplifier, the

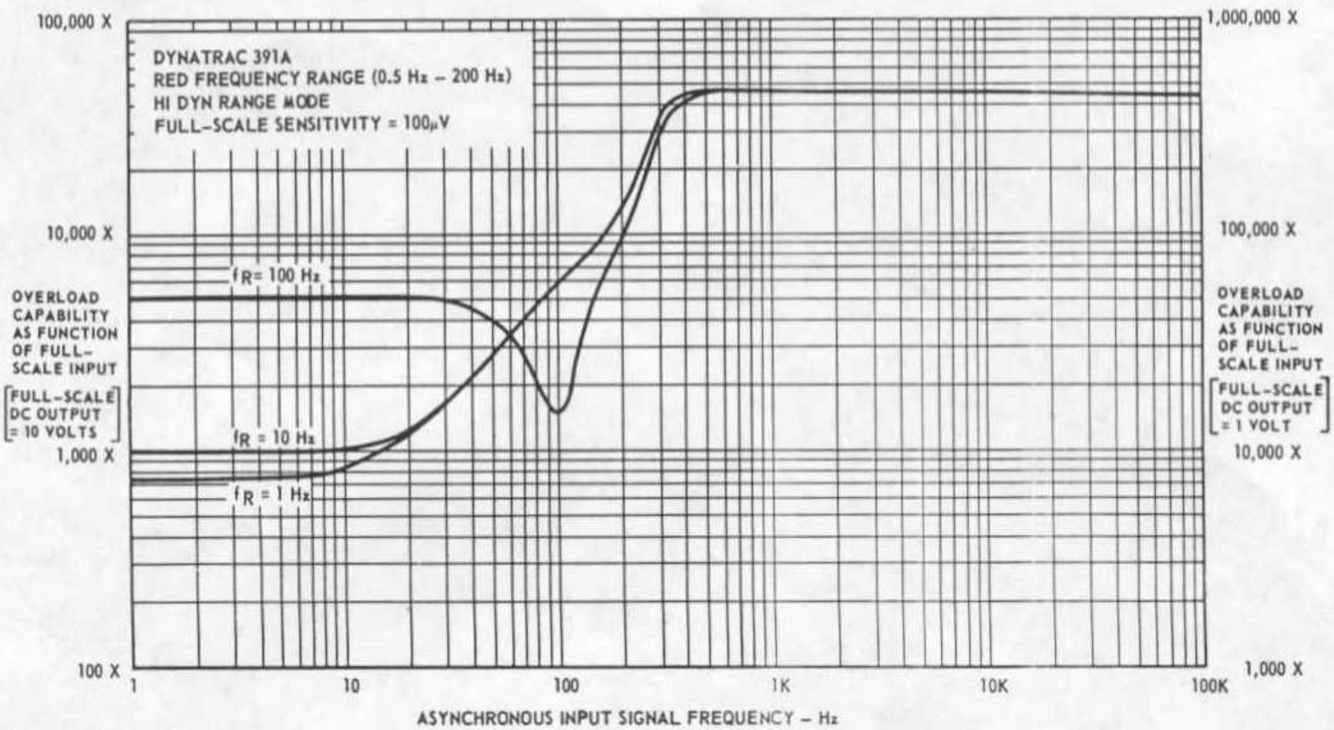


FIGURE 6 TYPICAL OVERLOAD CAPABILITY

SENSITIVITY DIAL SETTING	LO DRIFT			NORMAL			HI DYN RANGE		
	OVERLOAD CAPABILITY MIN	OVERLOAD CAPABILITY MAX	FULL-SCALE SENSITIVITY (rms)	OVERLOAD CAPABILITY MIN	OVERLOAD CAPABILITY MAX	FULL-SCALE SENSITIVITY (rms)	OVERLOAD CAPABILITY MIN	OVERLOAD CAPABILITY MAX	FULL-SCALE SENSITIVITY (rms)
100mV	4	4	1V	40	40	100mV	400	400	10mV
30mV	10	12	300mV	100	120	30mV	1000	1200	3mV
10mV	10	40	100mV	100	400	10mV	1000	4000	1mV
3mV	10	120	30mV	100	1200	3mV	1000	12000	.3mV
1mV	10	400	10mV	100	4000	1mV	1000	40000	.1mV
.3mV	10	400	3mV	100	4000	.3mV	1000	40000	30µV
.1mV	10	400	1mV	100	4000	.1mV	1000	40000	10µV
30µV	10	400	.3mV	100	4000	30µV	1000	40000	3µV
10µV	10	400	.1mV	100	4000	10µV	1000	40000	1µV
3µV	10	400	30µV	100	4000	3µV	1000	40000	300nV
1µV	10	400	10µV	100	4000	1µV	1000	40000	100nV

TABLE 2 VARIATION OF OVERLOAD CAPABILITY WITH SENSITIVITY

purpose of the front-end filtering is to improve the overload capability or dynamic reserve of the instrument (see appendices F and G). Figure 6 shows the overload capability of a DYNATRAC 391A equipped with a red card set (0.5Hz–200Hz). The curves shown were measured with a full-scale input of 100µVrms and at 3 different reference frequencies 1Hz, 10Hz and 100Hz. At each measured value of asynchronous frequency, the amplitude required to cause a 5% decrease in the full-scale output was recorded. Note that the overload capability is increased by a factor of 10 if a full-scale output of 1 volt is used instead of 10 volts. This '1 volt overload capability' corresponds to the overload capability specified for competing instruments with a full-scale output of only 1 volt. The overload capability of the DYNATRAC 391A with other card sets can be obtained from figure 6 by frequency scaling.

Table 2 shows how the overload capability and sensitivity of the DYNATRAC 391A varies with sensitivity dial setting and operating mode. The minimum and maximum overload values shown correspond to the low and high frequency ends of each frequency range, as shown in figure 6. The overload figures given in Table 2 are for a 10 volt full-scale output and should be multiplied by 10 if a 1 volt full-scale output is used.

Figure 7 shows the results of similar tests made on a DYNATRAC 391A equipped with an orange card set and on a (more expensive) competitive, manually-tuned instrument. In this case, a reference or synchronous frequency of 1kHz was used. Notice that the front-end filter in the competitive instrument is of limited value due to overloading at the filter input (see appendix F).

The DYNATRAC 391A includes nine overload detectors at critical points, throughout the signal path. Independent positive and negative detectors are used in order to insure detection of asymmetrical waveforms. An overload light on the front panel indicates any overload condition.

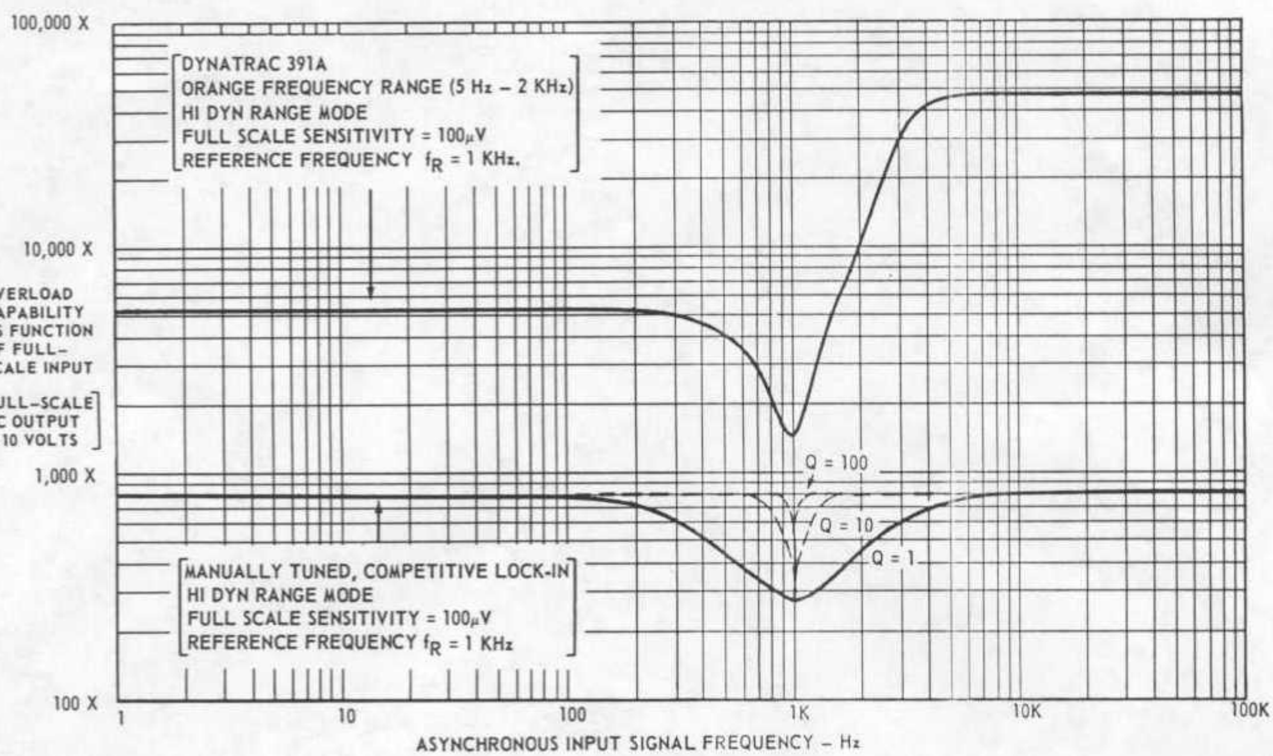


FIGURE 7 COMPARISON BETWEEN DYNATRAC 391A AND CONVENTIONAL INSTRUMENT

One significant advantage of the DYNATRAC system is the fact that harmonic relationships *do not exist* after signal mixing. A harmonic of the reference frequency f_r cannot be a harmonic of the IF frequency f_o after the signal mixer and will therefore not be detected by the synchronous detector. For example, let f_r = 100Hz and f_o = 5.5kHz, so that

$$f_v = f_o + f_r = 5.6\text{kHz}$$

Table 3 shows the output frequencies of the signal mixer for harmonics of 100Hz. Note that they are not harmonically related to the synchronous f_o frequency (5.5kHz). It is also important to note that this elimination of harmonic responses is not caused by the bandpass filter as is the case (with imperfect results) in a conventional tuned instrument, but is due to the heterodyning or frequency mixing technique.

The only possible response to a harmonic involves mixing a signal harmonic with the same harmonic of f_v as shown in figure 8.

The signal mixer products will be:

$$nf_v \pm nf_r = nf_o \quad \text{or} \quad n(f_o + 2f_r)$$

For odd values of n, nf_o would be detected by the PSD. However, the BP filter will effectively eliminate signals at frequencies of nf_o where $n > 1$.

With zero voltage at its signal input, the output of any phase-sensitive detector (PSD) will contain positive and negative 'spikes' due to capacitive coupling from the PSD square-wave, gating input. Any imbalance between the positive and negative 'spikes' will cause an output offset voltage which is frequency dependent. A change in signal frequency may therefore produce a change in output offset voltage. Since the PSD in the DYNATRAC 391A operates at a constant frequency (for each frequency range) this type of zero drift is eliminated and the output dc drift for the brown, red and orange card sets (0.1Hz–20Hz, 0.5Hz–200Hz and 5Hz–2kHz) is extremely low. (less than 5ppm/°C in LO DRIFT MODE)

Though often not specified, the output stability of all lock-in amplifiers degrades somewhat at higher frequencies due to the increasing effect with frequency of the finite rise and fall times of the gating waveforms used. That is even with a constant gating frequency,

f_s	SUM FREQUENCY $f_v + f_s$	DIFF. FREQUENCY $f_v - f_s$
100	5.7 kHz	5.5 kHz (= f_o)
200	5.8 kHz	5.4 kHz
300	5.9 kHz	5.3 kHz
400	6.0 kHz	5.2 kHz
500	6.1 kHz	5.1 kHz
600	6.2 kHz	5.0 kHz
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TABLE 3 HARMONIC RESPONSES

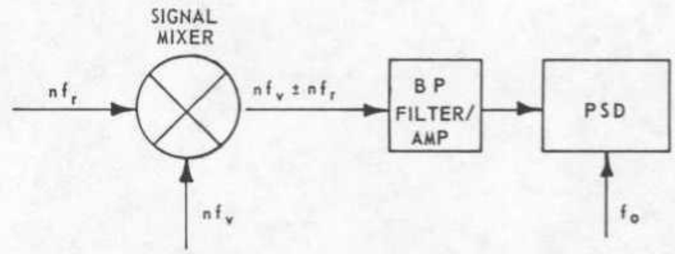

FIGURE 8 SIGNAL MIXER OPERATION

changes in waveform rise and fall times or semiconductor capacitance with temperature will cause an output instability or drift that increases with frequency. In non-heterodyning lock-ins, the output stability decreases proportionately with increasing frequency. In the DYNATRAC 391A, the output stability is independent of frequency within a given range, but decreases to 10ppm/°C for the yellow card set (50Hz–20kHz) and 100ppm/°C for the green card set (5kHz–200kHz).

A new current switching type of PSD is used in the DYNATRAC 391A and provides extremely high linearity – typical non-linearity is 0.02% (0.05% maximum).

3. SIGNAL MODES

The output dc amplifier is a two-pole (–12dB/octave) low-pass amplifier with switchable time-constant and gain. For use in feed back loops, the amplifier roll-off can be converted to –6dB/octave by means of a rear panel 6dB/12dB switch.

There are three switch selectable, signal measuring modes, HI DYN RANGE, NORMAL and LO DRIFT (see table 4). The gain of the output amplifier, and hence the the instrument sensitivity, is changed as the mode is changed, as indicated by the two light emitting diodes (LEDs) associated with the sensitivity switch. In the LO DRIFT mode, time-constant values are also reduced by a factor of 10 as indicated by the LED associated with the time-constant switch. This allows an increase in signal bandwidth for such experiments as Auger spectroscopy.

MODE	SENSITIVITY [1 – 3 – 10 SEQUENCE]	TIME CONSTANT [1.25 – 4 – 12.5 SEQUENCE]	NOISE BANDWIDTH [10 – 3.16 – 1 SEQUENCE]	ZERO SUPPRESS RANGE
LO DRIFT	10μV – 1V	.125ms (approx.) –12.5sec	1kHz (approx.) –0.01Hz	±10% FS
NORMAL	1μV – 100mV	1.25ms –125sec	100Hz –0.001Hz	±100% FS
HI DYN RANGE	100nV – 10mV	1.25ms –125sec	100Hz –0.001Hz	±1000% FS

TABLE 4 SIGNAL MODES

In all three signal modes, the full-scale dc output is 10 volts (front panel) and a variable 0–1 volt full-scale output is provided on the rear panel. As mentioned previously, the instrument can be operated with full-scale outputs of 1 volt (front panel) and 0–100mV (rear panel) to provide a 10 times increase in overload capability. However, as discussed in appendix F, the resulting (astronomic) values of overload capability are rarely if ever needed.

In the HI DYN RANGE mode, the gain of the output amplifier is at a maximum so that the required full-scale dc output signal from the PSD is at a minimum. The corresponding full-scale ac input signal to the PSD is therefore also at a minimum resulting in a maximum overload capability (see table 2 and appendix F).

In the NORMAL mode, the gain of the dc amplifier is reduced by a factor of 10 to give a 10 times decrease in both overload capability and dc drift. Similarly in the LO DRIFT mode, the gain of the dc amplifier is reduced by an additional factor of 10 to give a minimum overload capability and maximum output stability (minimum dc drift) as shown in table 5.

The purpose of the three signal modes is to allow a trade-off between output stability and overload capability. The NORMAL mode should be used if an overload is indicated in the LO DRIFT mode. Similarly, the HI DYN RANGE mode should only be used if overloading occurs in the NORMAL mode or if the additional factor of 10 increase in sensitivity is required.

FREQUENCY RANGE	HI DYN RANGE	NORMAL	LO DRIFT
BROWN, RED, ORANGE (0.1Hz – 2kHz)	500ppm/°C	50ppm/°C	5ppm/°C
YELLOW (50Hz – 20kHz)	1000ppm/°C	100ppm/°C	10ppm/°C
GREEN (5kHz – 200kHz)	10,000ppm/°C	1000ppm/°C	100ppm/°C

TABLE 5 OUTPUT DC DRIFT OF DYNATRAC 391A

4. NOISE MODE

In addition to the three signal modes described in section 3, an optional noise measuring mode is available (early DYNATRAC 391 models have a standard, less flexible NOISE mode). Such a NOISE mode is useful, for example, in measuring the spectral noise of photo detectors.

As discussed in the appendices to this note, a conventional broadband lock-in amplifier acts as a number of parallel bandpass filters with center-frequencies occuring at the harmonics of the reference signal to the instrument and which will track changes in the reference frequency. In a conventional instrument, a tunable front-end bandpass filter may be used to attenuate the harmonic responses but the instrument will then be unable to track changes in the reference frequency. In the DYNATRAC 391A, there are no such harmonic responses and the instrument acts as a single tracking bandpass filter centered at the reference frequency. In any of the three signal modes, any noise at the input of the instrument will be amplified and filtered and, being asynchronous, will produce an ac noise output (see appendix D). When fitted with the optional NOISE mode, an ac amplifier/rectifier/filter circuit is added to the output of the dc amplifier in the DYNATRAC 391A, so that the meter reads the input noise level and a noise analog dc voltage is provided at a rear panel BNC connector. A switch is also provided on the rear panel to select noise smoothing time constants of 1, 10 and 100 seconds.

By setting the time constant/bandwidth switch to the desired noise bandwidth, the DYNATRAC 391A can then be used as a fixed bandwidth variable frequency noise-meter. The frequency at which the input noise is measured is determined by the reference frequency and either the internal or an external reference may be used, so that swept frequency noise measurements are possible. For noise measurements, no setting of the phase controls is required so that it is only necessary to set the bandwidth, sensitivity and smoothing time-constant values.

FREQUENCY RANGE (Hz)	NOISE BANDWIDTH (Hz)	NOISE SMOOTHING (Seconds)
1K – 200K	100	1 (or 10)
100 – 1K	10	10 (or 100)
10 – 100	1	100
1 – 10	0.1	Use Normal mode and signal output. Chart recording should be used to average noise.
.1 – 1	0.01	

TABLE 6 SUGGESTED NOISE BANDWIDTH AND SMOOTHING VALUES

MODE	MULTIPLY SENSITIVITY DIAL SETTING BY	
	SIGNAL OUTPUT (Front Panel)	NOISE OUTPUT (Rear Panel)
LO DRIFT (10μV – 1V)	X10 *	X1
NORMAL (1μV – 100mV)	X1 *	X.1
HI DYN RANGE (100nV – 10mV)	X.1 *	X.01
NOISE (100nV – 10mV)	X1	X.1 *

*Correctly indicated by sensitivity switch setting and associated LED indicators

TABLE 7 SIGNAL AND NOISE SENSITIVITY WITH MODE SELECTION

Table 6 shows recommended values of noise bandwidth and smoothing time-constant settings for different frequency ranges. With the values shown, the rms fluctuation of the output voltage will be less than 2.5%. As discussed in appendix E, the actual bandwidth of the measured noise will be twice that indicated on the TIME-CONSTANT/BANDWIDTH switch.

The noise measuring circuitry is ac coupled to the dc amplifier output so that the noise circuitry will not respond to synchronous signals. Though the noise output is displayed on the meter only in the NOISE position of the mode switch, noise outputs are available at the rear panel output for HI DYN RANGE, NORMAL and LO DRIFT modes as shown in table 7. As can be seen from this table, it is possible to measure simultaneously and separately, a synchronous signal and the noise accompanying it.

5. REFERENCE CHANNEL

The reference channel circuitry supplies the required mixing frequencies for the signal mixer (f_v) and the PSD (f_o). In the INT reference mode, the reference channel also provides a variable amplitude, sinusoidal reference output signal whose frequency may be varied over the frequency range in use. In the EXT or 2f reference modes (see figure 9), the reference channel locks-on to an external reference signal.

In the EXT or 2f modes, the phase-detector has two inputs, the external reference signal of frequency f_r and an internally generated f_r signal. The phase-detector compares the phase of its two inputs and produces an output voltage proportional to their difference in phase. This output voltage is amplified and smoothed by the loop filter and used to control the frequency of a voltage controlled oscillator (VCO) of frequency f_v. The frequency f_v is mixed in the reference mixer with the output of a second oscillator operating at a frequency of f_o which is constant for each set of frequency range cards. The circuitry used for the f_o oscillator is almost identical to that used for the IF (f_o) bandpass filter. The f_o oscillator consists of a bandpass filter with a positive-feedback, amplitude limited loop around it to obtain a stable oscillation. In this way any slight drift of oscillator frequency f_o with temperature is matched by a corresponding change in the center-frequency f_o of the IF bandpass filter.

The reference mixer mixes the square-wave f_v signal from the VCO with the sine-wave f_o signal and the low-pass filter selects the sinusoidal difference frequency $f_v - f_o$. The two inputs to the phase-detector will therefore be f_r and $f_v - f_o$. The feedback loop comprising the phase-detector, loop-filter, VCO, reference mixer and low-pass filter will phase-lock the two inputs to the phase detector such that

$$f_r = f_v - f_o \qquad \text{or} \qquad f_v = f_o + f_r$$

This type of circuit is called a heterodyning phase-locked-loop (PLL). The phase-detector used includes a phase-compensation circuit that controls the phase-error caused by the two low-pass filters (in signal and reference channels). The internally generated reference also drives the CAL circuit which provides an accurate (1%) square-wave output signal for calibration purposes which has a 10mVrms fundamental frequency component. Unlike other lock-ins, the DYNATRAC 391A measures only the fundamental frequency component of its signal input and the CAL square-wave signal therefore has an rms amplitude of 11.1mV.

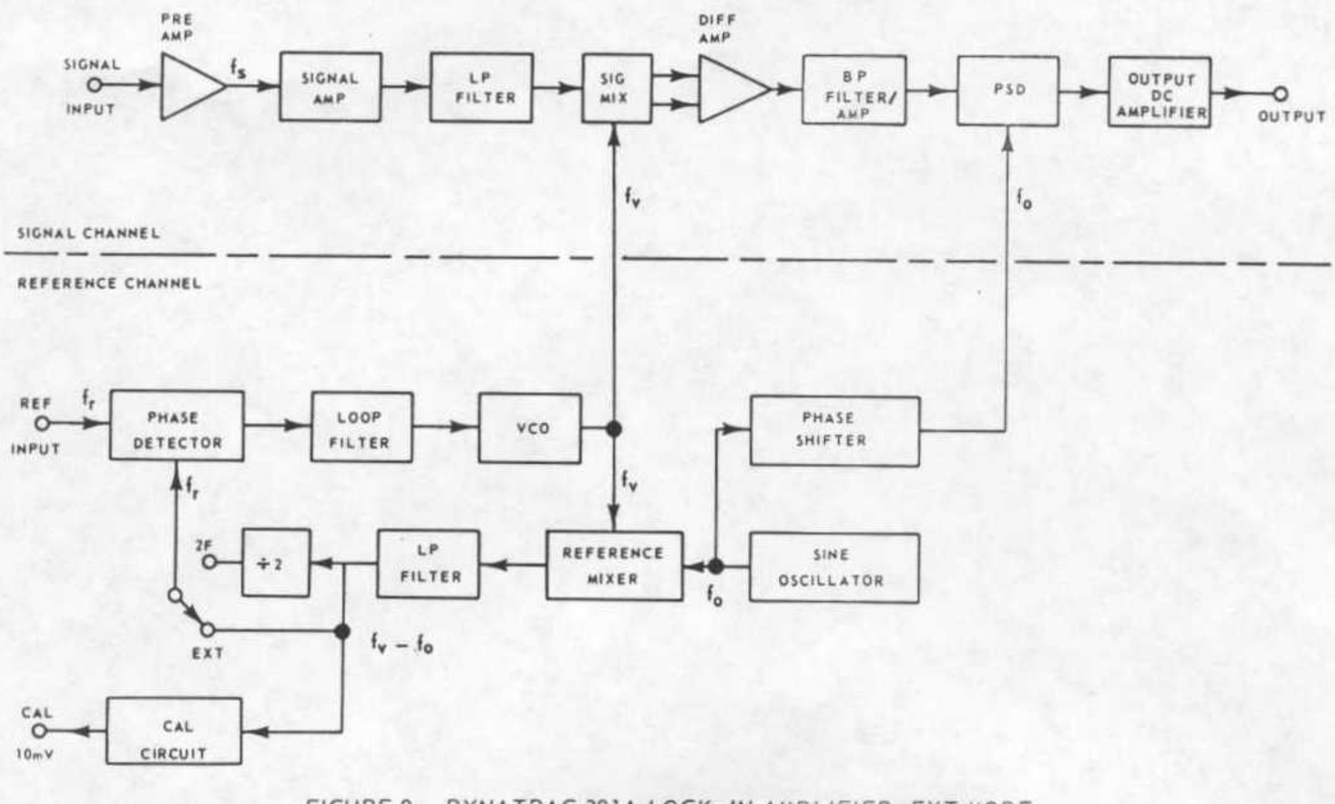

FIGURE 9 DYNATRAC 391A LOCK-IN AMPLIFIER, EXT MODE

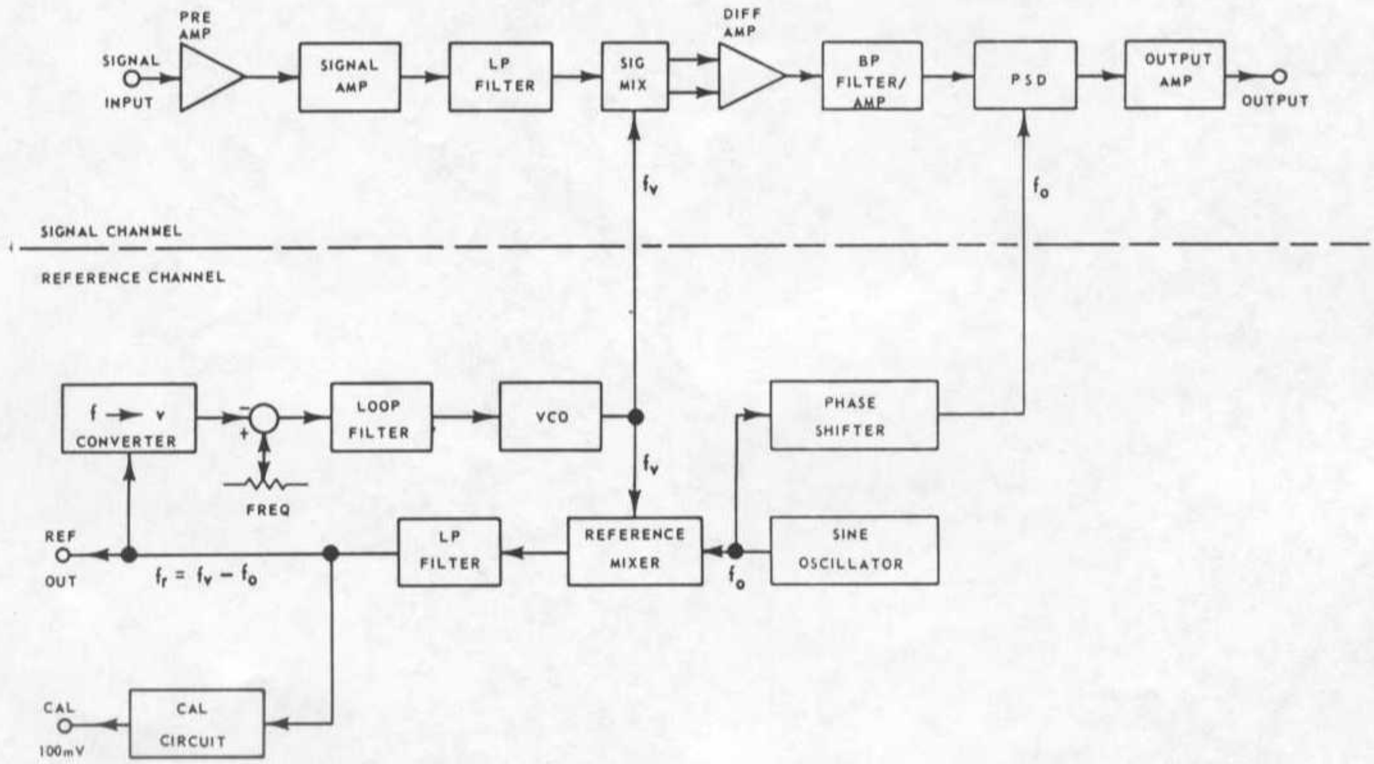

FIGURE 10 DYNATRAC 391A LOCK–IN AMPLIFIER, INT MODE

In the 2f mode, the internal reference frequency is divided by two before the phase-detector so that

$$f_r = \frac{f_v - f_o}{2} \qquad \text{and therefore}$$

$$2f_r = f_v - f_o \qquad \text{or} \qquad f_v = f_o + 2f_r$$

In this mode, the DYNATRAC 391A will detect and measure input signals at the second harmonic of its reference input ($2f_r$) and with excellent rejection of the fundamental frequency component (f_r).

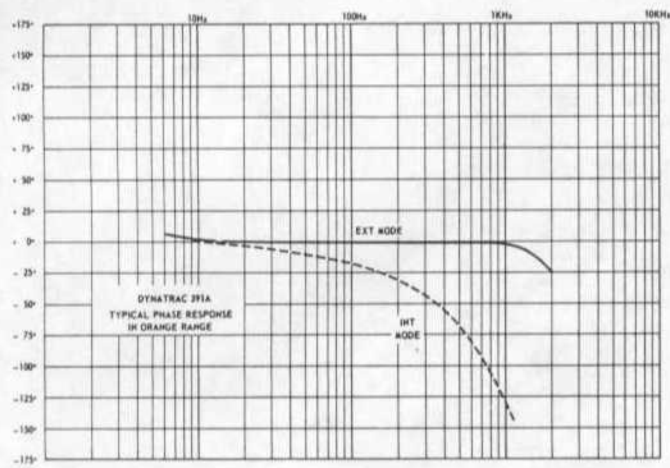

FIGURE 11 PHASE RESPONSE

In the EXT or 2f modes, a front panel light indicates when the phase-locked-loop is not locked due to a missing reference, out of range reference or because the loop has not yet settled.

Figure 10 shows the DYNATRAC 391A in the INT reference mode. In this mode, the phase detector is replaced by a frequency to voltage converter so that the reference channel generates its own reference. Both frequency and amplitude are adjustable by front panel controls. In the INT mode, the phase-compensation circuitry is bypassed so that the phase control must be readjusted for each change in frequency setting. Figure 11 shows typical phase responses for both EXT and INT reference modes.

In conventional lock-in amplifiers, the phase-shift circuitry must deal with a very wide range of frequencies. In the DYNATRAC 391A, the PSD and phase-shifter operate at the constant frequency f_o, allowing improved phase accuracy and stability.

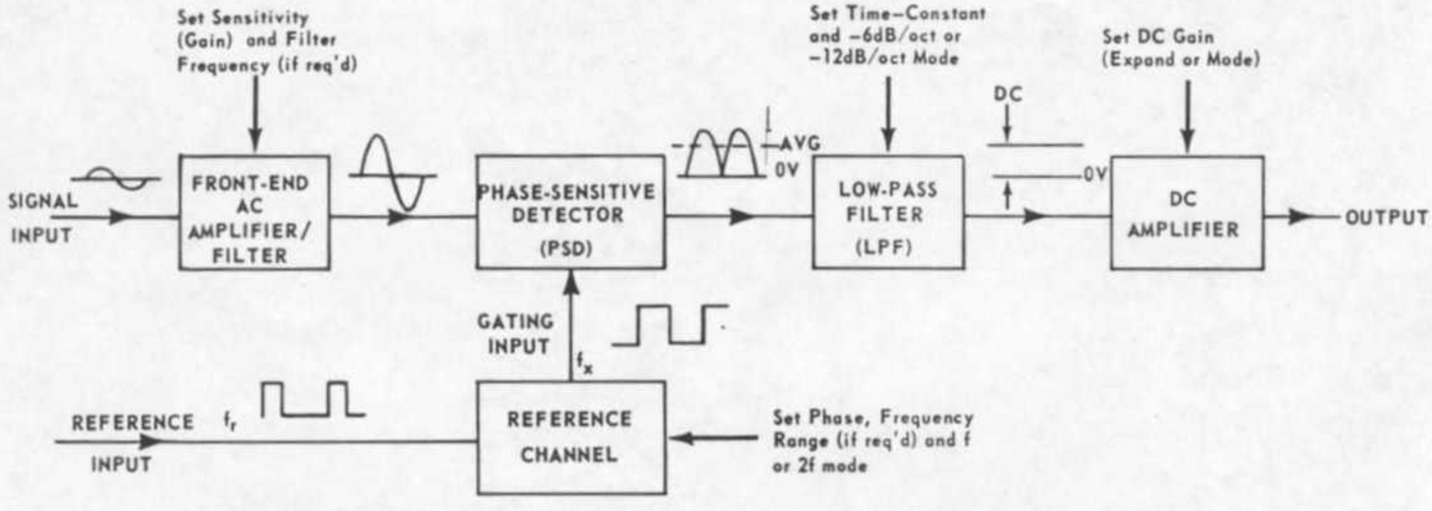

FIGURE A1 BASIC LOCK–IN AMPLIFIER

APPENDIX A – THE LOCK–IN AMPLIFIER

A lock-in 'amplifier' shown in basic form in figure A1, is an instrument which uses phase-sensitive detection, filtering and amplification in order to measure small ac signals. Unlike a conventional ac voltmeter, the lock-in amplifier "locks-in" to the frequency (f_r) of a reference signal and will measure input signals at that frequency only (or its odd harmonics).

The reference channel can accept a wide variation in frequency, amplitude and wave-shape (i.e. sine-wave, square-wave etc.) of the input reference signal. The reference channel output is the gating input to the phase-sensitive detector (PSD) circuit and is a constant amplitude, square-wave signal of frequency f_X, the phase of which can be continuously adjusted. In a non-heterodyning lock-in, $f_X = f_r$, the reference frequency. In the DYNATRAC 391A, $f_X = f_o$, the constant IF frequency.

The signal input to the lock-in is amplified to a level suitable for phase-sensitive detection. In many lock-ins, the gain of the front-end amplifier is frequency selective (i.e. it is also a filter) in order to improve its overload capability. In the DYNATRAC 391A the front-end amplifier also contains a mixing circuit which heterodynes signals at the reference frequency f_r up to a higher IF frequency f_o.

The output of the PSD circuit is the product of its two inputs (signal and gating). Any input signal of frequency f_X (i.e. synchronous signals) will produce a dc output, input signals at other frequencies (i.e. asynchronous signals) will produce ac output signals.

The low-pass filter (LPF) removes the unwanted ac output signals from the PSD and passes the wanted dc signal to the input of a dc amplifier which amplifies it to a suitable level for display or recording.

APPENDIX B – SIGNAL/NOISE IMPROVEMENT WITH A LOCK–IN AMPLIFIER

In many experiments, the noise accompanying the output signal will limit the signal resolution. It is therefore desirable to pass the noisy signal through a 'black-box' which will reduce the noise without reducing the signal. Depending upon the type of signal (narrow pulse, sine-wave, etc.) the most suitable 'black-box' may be a box-car integrator, signal-averager, computer, filter, lock-in amplifier, etc.

In a signal averager or computer, the noisy signal is continuously sampled, digitized and digitally stored. With repetitive signals, the stored signal will increase with time. Stored signals due to random noise will decrease with time since the noise will average to zero. The resulting improvement in signal to noise ratio is therefore a function of the measurement time.

With an electronic filter or a lock-in amplifier, the bandwidth of the 'black-box' can be decreased to a minimum compatible with the signal bandwidth. Only noise having frequencies within the effective noise bandwidth of the filter or lock-in amplifier will pass through to the output and under normal circumstances the signal to noise ratio will be improved. Narrowing the bandwidth of such a 'black-box' is however only obtained at the expense of increasing the time-constant or time required to measure a change in signal amplitude. The resulting improvement in signal to noise ratio is again a function of the measurement time.

Figure B1 shows an example of the use of a filter or lock-in amplifier in improving the signal to noise ratio of a signal. Chopped light falling on the photo-cathode of the photo-multiplier tube (PMT) creates an ac anode current and hence an ac signal voltage e_s is developed across the load resistor R_L. The total capacitance C_L across the resistor R_L due to stray and wiring capacitances determines the –3dB signal bandwidth f_c.

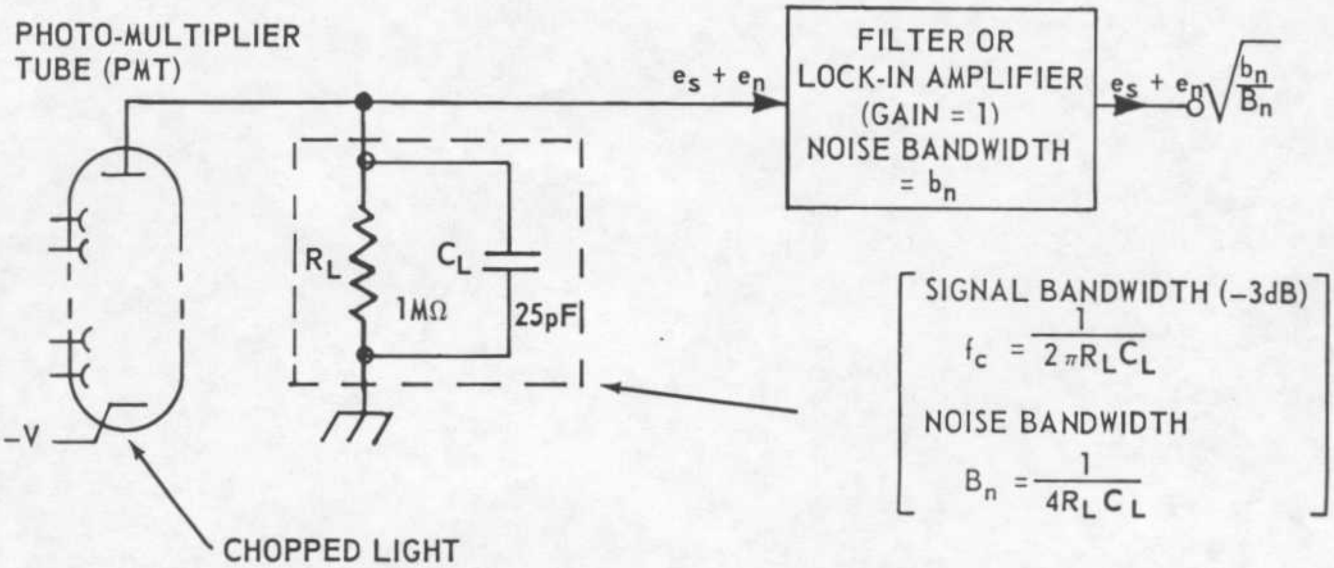

FIGURE B1 SIGNAL/NOISE IMPROVEMENT OF PMT OUTPUT

That is $f_c = \frac{1}{2\pi R_L C_L}$

The equivalent noise bandwidth B_n of the PMT output circuit will therefore be

$$B_n = \frac{\pi}{2} \times f_c = \frac{1}{4R_L C_L}$$

Suppose R_L = 1MΩ and C_L = 25pF, then

$$B_n = \frac{1}{4 \cdot 10^6 \cdot 25 \cdot 10^{-12}} = \underline{10\text{kHz}}$$

Due to the shot noise generated by the PMT and the noise generated by the resistor R_L, a noise voltage e_n will also be developed across R_L.

For simplicity, assume that the noise is evenly distributed across the noise bandwidth B_n (i.e. white noise). The signal to noise ratio at the input to the lock-in amplifier or filter will be $\frac{e_s}{e_n}$. If the effective noise bandwidth of the lock-in or filter is b_n, then the output signal to noise ratio will be

$$\frac{e_s}{e_n\sqrt{\frac{b_n}{B_n}}} = \frac{e_s}{e_n}\sqrt{\frac{B_n}{b_n}} \quad \text{(assuming unity gain)}$$

If b_n is say 0.01Hz, then the improvement in signal to noise ratio will be

$$\sqrt{\frac{B_n}{b_n}} = \sqrt{\frac{10^4}{10^{-2}}} = 1000$$

The advantage of a lock-in amplifier over a conventional filter is that its effective noise bandwidth b_n can normally be made orders of magnitude smaller than that of a filter.

The minimum detectable signal or resolution of a lock-in amplifier is dependent upon a number of factors. In the ideal case of an input signal having no noise associated with it, random or discrete, the resolution will be limited by either

a) the output noise generated within the effective noise bandwidth of the instrument by the front-end ac amplifier and/or an associated remote preamplifier

OR

b) the output zero uncertainty caused by noise and offset voltage drift in the output dc amplifier.

With a signal accompanied by noise, the overload capability of the instrument becomes important and any non-linearities in the signal channel may limit the resolution of an instrument. With random white noise interference, the resolution or signal/noise ratio can never be better than that predicted by theory using the square root of the noise bandwidth ratio as described above. In practice, the resolution is usually limited by a combination of these factors.

APPENDIX C – THE LOW–PASS FILTER

The type of output low-pass filter used in lock-in amplifiers consists of one or two resistor-capacitor networks as indicated in figure C1 and each network has associated with it, a time-constant of RC seconds (for R in ohms and C in farads). For a single RC section, the frequency response of the filter will roll off at −6dB/octave (i.e., the output will be halved for each doubling of frequency), from a −3dB corner frequency of $f_c = \frac{1}{2\pi RC}$. The equivalent noise bandwidth i.e. a rectangular bandwidth having the same area and maximum gain, will be $b_n = f_c \cdot \frac{\pi}{2} = \frac{1}{4RC}$. With two equal and independent RC networks,

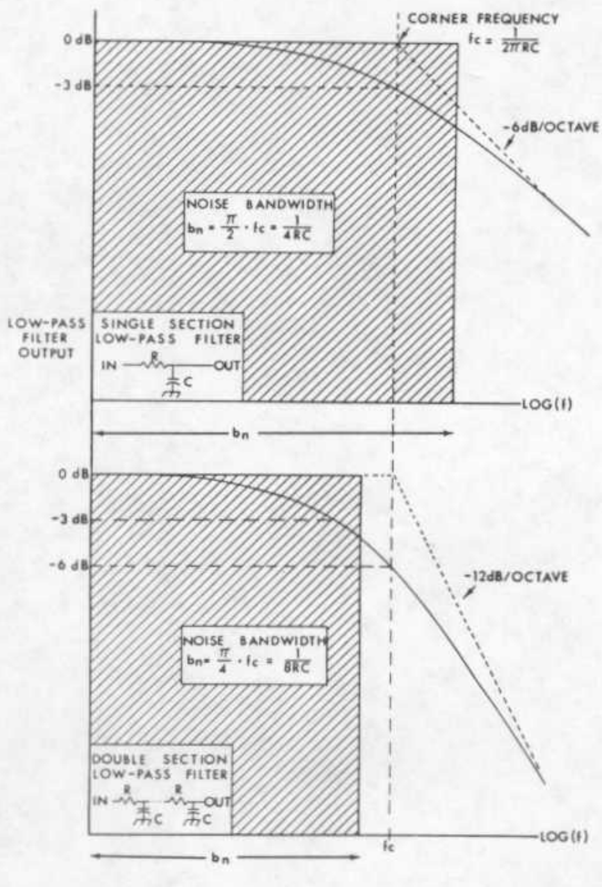

FIGURE C1 FREQUENCY RESPONSE OF A LOW–PASS FILTER

the frequency response of the filter will roll off at −12dB/octave (i.e., the output will be quartered for each doubling of frequency), from a −6dB corner frequency of $f_c = \frac{1}{2\pi RC}$ and the noise bandwidth will be $b_n = f_c \cdot \frac{\pi}{4} = \frac{1}{8RC}$. For such a double section filter, the response falls by 3dB (to 70%) at a frequency of $\frac{1}{9.76RC}$.

At first sight, it might appear that provision of a −12dB/octave mode merely allows the noise bandwidth to be reduced to one-half of that resulting from a −6dB/octave mode. When an interfering signal having a discrete frequency is encountered however, the additional rejection of such a signal provided by the −12dB/octave mode can be invaluable. For example, suppose the input to the low-pass filter from the PSD contains a strong interfering signal at a frequency of say $8f_c$ i.e., 3 octaves above f_c. In the −6dB/octave mode, this signal will be attenuated by approximately 18dB or 8 times. In the −12dB/octave mode however, the same signal will be attenuated by approximately 36dB or 64 times.

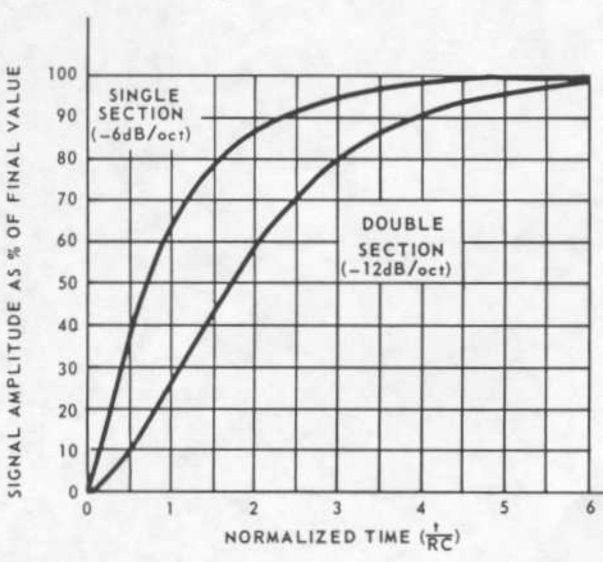

FIGURE C2 LOW–PASS FILTER RESPONSE TO STEP INPUT

A −6dB/octave mode is useful when the entire lock-in amplifier is to be used as a component in a feedback loop. In such a case, a −12dB/octave mode may cause instability.

The 'time-constant' of a lock-in is sometimes mistakenly assumed to be the measurement time. For a step change in the input ac signal to a lock-in, the measurement time depends both on the time constant selected and the required measurement accuracy. Figure C2 shows the relationship between measurement time (normalized to the time-constant value) and the output signal as a percentage of its final value. For the output to rise to within 5% of its final value will take 3.0 and 4.7 time-constants in the −6dB/octave and −12dB/octave modes respectively - an accuracy of within 1% will require 4.6 and 6.6 time-constants respectively. Note that for a given noise bandwidth, the measurement time for a −6dB/octave mode will be longer than for a −12dB/octave mode.

APPENDIX D – PHASE–SENSITIVE DETECTION AND THE 'SYNCHRONOUS FILTER'

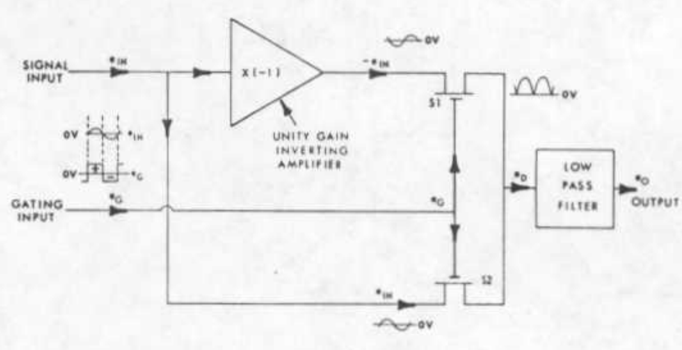

FIGURE D1 PHASE SENSITIVE DETECTOR

a) FUNDAMENTALS

A phase-sensitive or synchronous detector is a multiplier. As shown in simplified form in figure D1, any PSD has two inputs and one output. The two inputs consist of the signal e_{IN} whose synchronous component is to be detected and a gating signal e_G. The output of the PSD, e_D, is the product of its two inputs.

i.e. $e_D = e_{IN} \cdot e_G$

The gating signal e_G consists of a voltage squarewave centered about zero, which when it is positive (say), causes the semiconductor switch S2 to close and connect the unchanged input signal to the PSD output. When e_G is negative, switch S1 closes and the inverted signal is connected to the PSD output. In a lock-in amplifier, the phase of the gating squarewave can be adjusted to be in phase with the synchronous component of the input signal by using the phase controls to peak the instrument (and therefore the PSD) output. The gating signal, after phasing, can therefore be described by

$$e_G = \frac{4}{\pi}[\sin\omega_X t + \frac{1}{3}\sin 3\omega_X t + \frac{1}{5}\sin 5\omega_X t + ---]$$

where $\omega_X = 2\pi f_X$ and where for a conventional lock-in $f_X = f_r$, i.e. the fundamental frequency of the reference signal to the instrument. For the DYNA-TRAC 391A, $f_X = f_o$ the IF frequency.

The input signal e_{IN}, normally consists of a synchronous component e_S (i.e. phase-locked to the gating signal) and an asynchronous component e_N due to random noise or discrete frequency interference (e.g. 60Hz power line pick-up) or both.

i.e. $e_{IN} = e_S + e_N$

For simplicity, let e_S be a simple sinusoidal signal

i.e. $e_S = E_S \sin\omega_X t$

and let e_N also be a simple asynchronous sinusoidal signal

i.e. $e_N = E_N \sin\omega_N t$

Then the PSD output signal e_D will be given by

$$e_D = e_{IN} \cdot e_G = e_S \cdot e_G + e_N \cdot e_G$$

$$\therefore e_D = \frac{4}{\pi}E_S \sin\omega_X t[\sin\omega_X t + \frac{1}{3}\sin 3\omega_X t + \frac{1}{5}\sin 5\omega_X t + ---]$$
$$+ \frac{4}{\pi}E_N \sin\omega_N t[\sin\omega_X t + \frac{1}{3}\sin 3\omega_X t + \frac{1}{5}\sin 5\omega_X t + ---]$$

Then $e_D = \frac{4}{\pi}E_S[\sin^2\omega_X t + \frac{1}{3}\sin\omega_X t \cdot \sin 3\omega_X t + \frac{1}{5}\sin\omega_X t \cdot \sin 5\omega_X t + ---]$ (DC + AC terms)

$+ \frac{4}{\pi}E_N[\sin\omega_N t \cdot \sin\omega_X t + \frac{1}{3}\sin\omega_N t \cdot \sin 3\omega_X t + \frac{1}{5}\sin\omega_N t \cdot \sin 5\omega_X t + ---]$ AC terms only (average value = 0)

Now $\sin A \cdot \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$

so that the average or dc value of e_D is simply

(DC term) (AC term)

$$\overline{e_D} = \frac{4}{\pi}E_S \cdot \overline{\sin^2\omega_X t} = \frac{4}{\pi}E_S \overline{\frac{1}{2}[1 - \cos 2\omega_X t]} = \frac{2}{\pi}E_S$$

The PSD output therefore will consist of a wanted dc signal and unwanted ac signals. A low-pass filter is used after the PSD to remove the unwanted ac signals and pass the dc signal.

When the phase difference ϕ, between the gating and synchronous signal input to the PSD is not zeroed, then e_S can be described by

$$e_S = E_S \sin(\omega_X t + \phi)$$

and the dc value of the PSD output will be given by

$$\overline{e_D} = \frac{4}{\pi}E_S\overline{[\sin(\omega_X t + \phi)\sin\omega_X t]}$$
$$= \frac{4}{\pi}E_S \cdot \frac{1}{2}\overline{[\cos\phi - \cos(2\omega_X t + \phi)]}$$
$$\therefore \overline{e_D} = \frac{2}{\pi}E_S\cos\phi$$

Notice that when $\phi = \frac{\pi}{2}$ (or $\frac{n\pi}{2}$, n odd), then

$$\overline{e_D} = \frac{2}{\pi}E_S\cos\frac{\pi}{2} = 0$$

With a high quality lock-in such as the DYNATRAC 391A, which has a phase quadrature (i.e. 0°, 90°, 180°, 270°) switching accuracy of ±0.2°, the phase controls can be set very accurately by adjusting the phase controls for zero output and then switching the phase by 90°.

The slope of the phase sensitivity of a PSD is given by

$$\left|\frac{d\overline{e_D}}{d\phi}\right| = \frac{2}{\pi}E_S\sin\phi$$

and $\sin\phi$ has a maximum value when $\phi = \frac{\pi}{2}$, so that adjusting for an output zero (i.e. $\phi = \frac{\pi}{2}$) provides the most sensitive means of setting phase. Typical PSD waveforms are shown in figure D2.

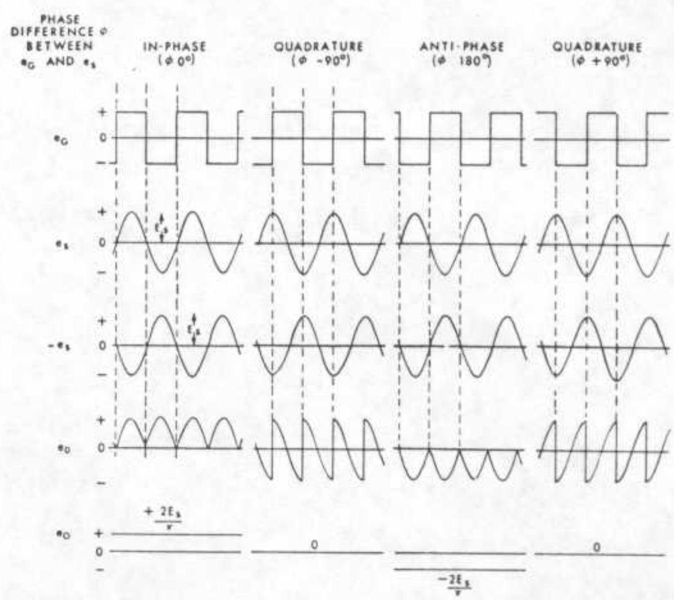

FIGURE D2 PSD WAVEFORMS

b) HARMONIC RESPONSES

If the synchronous component e_S of the PSD input is at a harmonic of the reference frequency f_X, then

$$\overline{e_D} = \overline{e_G \cdot e_S} = \overline{e_G \cdot e_S \sin n\omega_X t} \quad \text{and}$$

$$\overline{e_D} = \frac{4E}{\pi}S\overline{[\sin n\omega_X t \cdot \sin\omega_X t + \frac{1}{3}\sin n\omega_X t \cdot \sin 3\omega_X t}$$
$$\overline{+\frac{1}{5}\sin n\omega_X t \cdot \sin 5\omega_X t + ---]}$$

if n is odd, i.e. if n = 1, 3, 5, 7, 9, 11 etc., then one of the products in the above expression will have a dc component.

for example, if n = 3, then

$$\overline{e_D} = \frac{4E}{\pi}S \quad \overline{[\frac{1}{3}\sin^2 3\omega_X t]} = \frac{1}{3}\cdot\frac{2E}{\pi}S$$

if n = 7, then

$$\overline{e_D} = \frac{4E}{\pi}S\overline{[\frac{1}{7}\sin^2 7\omega_X t]} = \frac{1}{7}\cdot\frac{2E}{\pi}S$$

If n is even, there will be no dc output from the PSD.

c) SQUARE-WAVE INPUTS

If the synchronous signal input e_S to a PSD is a square-wave of peak value E_S, then

$$e_S = E_S \cdot \frac{4}{\pi}[\sin\omega_X t + \frac{1}{3}\sin 3\omega_X t + \frac{1}{5}\sin 5\omega_X t + ---]$$

and as before

$$\overline{e_D} = \overline{e_S \cdot e_G} = E_S \cdot \overline{\frac{16}{\pi^2}[\sin\omega_X t + \sin 3\omega_X t}$$
$$\overline{+ \sin 5\omega_X t + ---]^2}$$

so that $\overline{e_D} = E_S \cdot \frac{16}{\pi^2}[\frac{\pi^2}{16}] = E_S$

d) GAIN

A PSD circuit combined with a low-pass filter has a transfer function or gain which is different for synchronous and asynchronous signals.

For a synchronous sinusoidal input signal to a PSD given by

$$e_S = E_S \sin\omega_X t$$

the PSD output will be

$$e_D = \frac{2E}{\pi}S + \text{(ac components)}$$

The output e_f of the low-pass filter following the PSD will be a dc voltage of

$$e_f = \frac{2E_S}{\pi}$$

We may define the 'gain' of the combined PSD and low-pass filter circuits as

$$\text{gain} = \frac{\text{rms output from LP filter}}{\text{rms input to PSD}}$$

so that the SYNCHRONOUS GAIN is given by

$$\text{Synchronous Gain} = \frac{e_{f(rms)}}{e_{S(rms)}} = \frac{\frac{2E_S}{\pi}}{\frac{E_S}{\sqrt{2}}} = \sqrt{2}\cdot\frac{2}{\pi} \ (= 0.9)$$

For an asynchronous sinusoidal input signal to a PSD given by

$$e_N = E_N \sin\omega_N t$$

the PSD ac output (there will be no dc output) will be

$$e_D = \frac{4E}{\pi}N[\sin\omega_N t \cdot \sin\omega_X t + \frac{1}{3}\sin\omega_N t \cdot \sin 3\omega_X t$$
$$+\frac{1}{5}\sin\omega_N t \cdot \sin 5\omega_X t + ---]$$

and therefore

$$e_D = \frac{2E}{\pi}N[\cos(\omega_X - \omega_N)t - \cos(\omega_X + \omega_N)t$$
$$+\frac{1}{3}\cos(3\omega_X - \omega_N)t - \frac{1}{3}\cos(3\omega_X + \omega_N)t$$
$$+\frac{1}{5}\cos(5\omega_X - \omega_N)t - \frac{1}{5}\cos(5\omega_X + \omega_N)t + ---] \quad ①$$

The time-constant (RC) of the low-pass filter is normally set so that the filter will only pass very low frequencies. Only the $(\omega_X - \omega_N)$ frequency component in the above expression for e_D will pass through the low-pass filter and only then if $(\omega_X - \omega_N) \leq \omega_c$ where

$$\omega_c = 2\pi f_c$$

and f_c is the −3dB (i.e. 70%) cut-off frequency of the low-pass filter and

$$f_c = \frac{1}{9.76RC} \qquad \text{(see appendix C)}$$

For $(\omega_X - \omega_N) < \omega_c$ then, the output e_f of the low-pass filter will be

$$e_f = \frac{2E}{\pi}N\cos(\omega_X - \omega_N)t$$

which has an rms value of $\frac{\sqrt{2}E_N}{\pi}$

The ASYNCHRONOUS GAIN will therefore be given by

$$\text{Asynchronous Gain} = \frac{e_{f(rms)}}{e_{N(rms)}} = \frac{\frac{\sqrt{2}E_N}{\pi}}{\frac{E_N}{\sqrt{2}}} = \frac{2}{\pi}$$

The gain of the combined PSD and low-pass filter circuit will therefore be $\sqrt{2}$ (or 3dB) higher for synchronous signals than for asynchronous signals.

e) SYNCHRONOUS FILTER

If the synchronous frequency f_X of the gating signal to a PSD is 100Hz for example and the asynchronous input signal has a frequency f_N = 99Hz, then the frequencies in the PSD output (see equation ①) will be 1, 199, 201, 399, 401, 599Hz, etc.

Similarly, if the frequency of the asynchronous input signal changes to 101Hz with the same value of f_X = 100Hz, then the frequencies in the PSD output will be 1, 201, 199, 401, 399Hz etc. The output of the combined PSD and low-pass filter circuits will therefore be identical for both 99Hz and 101Hz input signals.

More generally, we can say that the PSD + filter circuit provides a symmetrical response centered about the synchronous frequency f_X and its odd harmonics. That is, input signals of frequencies δf above or below nf_X will produce a δf output from the low-pass filter. This type of frequency response has a *bandpass* characteristic and the bandpass filter formed by the combined PSD and low-pass filter circuits is often called a 'lock-in filter' or 'synchronous filter' or 'coherent filter'.

From the foregoing we can summarize the performance of a synchronous filter.

1. A synchronous filter has bandpass responses centered at the synchronous frequency f_X and all of its odd harmonics.

2. A synchronous filter will produce a dc output for synchronous input signals, i.e. of frequency f_X, $3f_X$, $5f_X$ etc.

3. The dc output will be proportional to cos ϕ where ϕ is the phase angle between the gating and input signals.

4. A synchronous filter will produce an ac output for asynchronous input signals.

5. The bandwidth of each bandpass response will be twice that of the low-pass filter, i.e. the low-pass filter response and its mirror image are centered at f_X, $3f_X$, $5f_X$ etc. (see figure D3).

6. The 'gain' of a synchronous filter is −3dB (i.e. 70%) for asynchronous signals (including random noise) compared to that for synchronous signals.

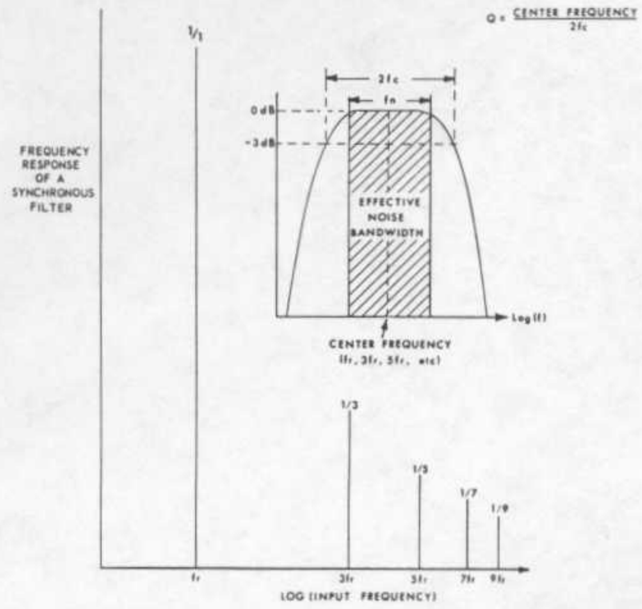

FIGURE D3 FREQUENCY RESPONSE OF A SYNCHRONOUS FILTER

7. The Q of a lock-in filter may be defined as for other bandpass filters i.e.

$$Q = \frac{\text{CENTER FREQUENCY } (nf_X)}{-3\text{dB BANDWIDTH } (2f_c)}$$

Unlike conventional bandpass filters, the Q of a synchronous filter can be as high as 2 x 10⁸.

APPENDIX E – NOISE BANDWIDTH AND NOISE MEASUREMENT

As mentioned in appendix D, the equivalent signal bandwidth ($2f_c$) and noise bandwidth ($2b_n$) of a synchronous filter is twice that of the low-pass filter (f_c and b_n). With normal time-constant settings, the noise bandwidth of the synchronous filter is also the noise bandwidth of the complete lock-in amplifier (see appendix G). The sensitivity or gain of the lock-in amplifier will however be $\sqrt{2}$ times (or 3dB) higher for a synchronous signal than for an asynchronous signal such as random noise.

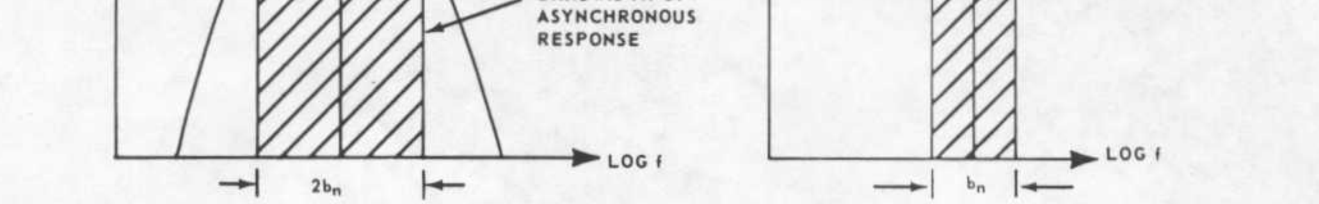

FIGURE E1 ACTUAL AND EQUIVALENT NOISE BANDWIDTHS

In normal use, a lock-in is used to measure small signals accompanied by noise, and for calculations of signal to noise improvement such as the example given in appendix B, it is much more convenient to assume that the gain of the lock-in is identical for signal and noise. The rms amplitude of a white noise signal is proportional to the square root of its noise bandwidth so that doubling the noise bandwidth will increase the noise output by 3dB. The rms noise output from a lock-in amplifier of actual bandwidth $2b_n$, will therefore be the same as that through a filter of noise bandwidth b_n and having identical gain to signal and noise (see figure E1).

With a low-pass filter with −12dB/octave roll-off, the noise bandwidth b_n is given by

$$b_n = \frac{1}{8RC}$$

The time-constant (RC) values used in the DYNATRAC 391A have been chosen so as to give simple values of b_n.

For example, if RC = 1.25 seconds, then

$$b_n = \frac{1}{8 \times 1.25} = 0.1\text{Hz}$$

In addition to time-constant values, the time-constant switch used on the DYNATRAC 391A also indicates equivalent values of noise bandwidth b_n.

When using the DYNATRAC 391A in its NOISE mode however, it must be remembered that the actual noise bandwidth of the instrument is $2b_n$, i.e. twice the indicated value.

APPENDIX F – THE OVERLOAD vs STABILITY TRADE-OFF

The overload capability of a lock-in amplifier (sometimes called dynamic reserve) is an important parameter since it is a measure of how much noise or interference (i.e. asynchronous signals) the instrument can tolerate when measuring a synchronous signal.

There will be a positive and negative value of peak signal input to the synchronous filter beyond which its phase-sensitive detector circuit will overload or saturate and cause gross output errors. For convenience, the overload level is usually defined in terms of the maximum *sinusoidal* signal e_{ov} that can be tolerated. For the purposes of this discussion, assume that the overload level is 10 volts rms, corresponding to peak values of ±14.14 volts for a sine-wave input signal.

Suppose for example, that the full-scale sinusoidal, synchronous input to the phase-sensitive detector is 1 volt rms (i.e. 1.414 volts peak). This will produce a dc output from the synchronous filter of $1.414 \cdot \frac{2}{\pi} = 0.9$ volts (see appendix C).

If the full-scale dc output is to be 10 volts, then the gain required from the dc amplifier is

$$G_{dc} = \frac{10}{0.9} = 11.1$$

Suppose further, that the voltage drift of the dc amplifier referred to its input is (say) 9µV/°C. The output drift or stability will therefore be $9G_{dc}$ µV/°C = 100µV/°C which can also be expressed in terms of the full-scale output as 10ppm/°C or 0.001%/°C.

The overload capability of this arrangement will be the overload level divided by the full-scale synchronous signal level

$$\text{i.e. Overload Capability} = \frac{10\text{V(rms)}}{1\text{V(rms)}} \text{ or } \frac{14.14\text{V (peak)}}{1.414\text{V (peak)}}$$

$$\text{or } \frac{28.28\text{V (peak-peak)}}{2.828\text{V (peak-peak)}} = 10$$

Note that the overload capability is *not* given by

$$\text{Overload Capability} = \frac{28.28\text{V}}{1\text{V}} \frac{\text{(peak-peak)}}{\text{(rms)}} = 28.28$$

Such a definition is an example of the 'specmanship' that has plagued lock-in amplifier data sheets in the past.

The terms 'overload capability' (dynamic reserve) and 'dynamic range' are often confused. The input dynamic range of a lock-in amplifier is defined as the overload level divided by the minimum detectable signal (see appendix B). The input dynamic range is therefore always larger than the overload capability or dynamic reserve. The output dynamic range of a lock-in is the full-scale dc output divided by the minimum detectable output signal. The dynamic reserve or overload capability is the difference between the input and output dynamic ranges – hence the term dynamic 'reserve'.

In many experiments, an overload capability of 10 is insufficient. A high quality lock-in such as the DYNATRAC 391A, is therefore provided with the capability of trading output stability for overload capability.

If the gain of the output dc amplifier is increased by a factor of 10, to G_{dc} = 111, then since

$$G_{dc}(111) = \frac{\text{FULL-SCALE OUTPUT (10V)}}{\text{FULL-SCALE INPUT}}$$

the full-scale input is now 90mV which corresponds to an rms sine-wave signal of 100mV at the input to the phase sensitive detector.

The overload capability is now given by

$$\text{Overload Capability} = \frac{10\text{V(rms)}}{100\text{mV(rms)}} = 100$$

i.e. increased by a factor of 10. The output stability will now be

$9G_{dc}$µV/°C = 1000µV/°C or 100ppm/°C or 0.01%/°C

Similarly, the gain of the output dc amplifier can be increased by a further factor of 10 to give an overload capability of 1000 and an output stability of 1000ppm/°C. In the DYNATRAC 391A, these three gain modes (from lowest dc gain to highest) are called LO DRIFT, NORMAL and HI DYN RANGE respectively.

High-performance lock-in amplifiers normally provide a 10 volt full-scale output – a full-scale of 1 volt is common for cheaper instruments. It is of interest to note that a 1 volt full-scale output may also be used with an instrument having a 10 volt full-scale capability. In this case, both the overload capability and output dc drift of the instrument will be increased by a factor of 10.

In the above discussion; it is assumed that the ac signal level (including noise) is at its highest at the input to the phase-sensitive detector. This is true if the ac front-end amplifier is wide-band, i.e. has no effective filtering. If filtering is used (see appendix G), it is possible that the filter input circuit may overload before an overload level is reached at the phase-sensitive detector. It is important therefore, that all possible overload points be monitored to prevent unknown overload conditions.

If a front-end filter is used, it is important to separate "in-band overload" from "out-band overload". The in-band overload capability of such an instrument is its overload capability in dealing with an interfering signal, close enough in frequency to the synchronous frequency, to be unaffected (i.e. not attenuated) by the filter, e.g. an interfering signal at 990Hz when the reference frequency is 1kHz. The out-band overload capability is simply the in-band overload capability plus the filter attenuation and is limited by the finite overload capability at the filter input.

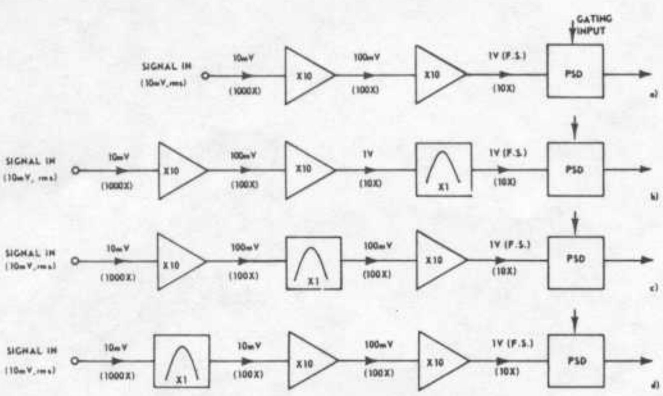

FIGURE F1 ALTERNATIVE POSITIONS OF FRONT–END FILTER

Figure F1 shows a simplified block diagram of the front-end of a lock-in amplifier. In figure F1a, the front-end consists simply of two amplifying stages each with a gain of 10. The full-scale (FS) input signal to the PSD is assumed to be 1 volt rms and the overload level at the input to all circuits (PSD, amplifiers and filter) is assumed to be 10Vrms or ±14.14 volts. In figure 1a then, the overload capability at the input to the first amplifier will be 1000 x FS, at the input to the second amplifier it will be 100 x FS and at the PSD input it will be 10 x FS. As the weakest link in the chain, the overload capability of an instrument using such a front-end would be that of the PSD input, i.e. 10 x FS as shown in figure F2a.

In figures F1b, c, and d, a bandpass filter of $Q = 10$ has been added to the front-end. The filter center frequency is assumed to be tuned to a reference frequency of 1kHz as shown in figure F2. In the position shown in figure F1b, the input overload capability of the filter is identical to that of the PSD and the filter provides no additional overload capability to the front-end.

As shown in figures F1c and d, the higher the gain after the filter, the higher is the overload capability of the front-end with the optimum filter position being at the input to the instrument. However, filters are inherently more noisy than amplifiers and the self-noise of an instrument is extremely important when dealing with 'clean' signals. For this reason, the position of a filter in the front-end of a lock-in amplifier of this type is always a compromise and changes with a change in sensitivity. In general then, the out-band overload capability that might be expected of such front-end filters (see dotted curve in figure F2) is not realizable in practice. It is essential that the PSD of a lock-in be designed for the highest possible overload capability – older lock-in designs have relied on front-end filters to hide poor PSD overload characteristics.

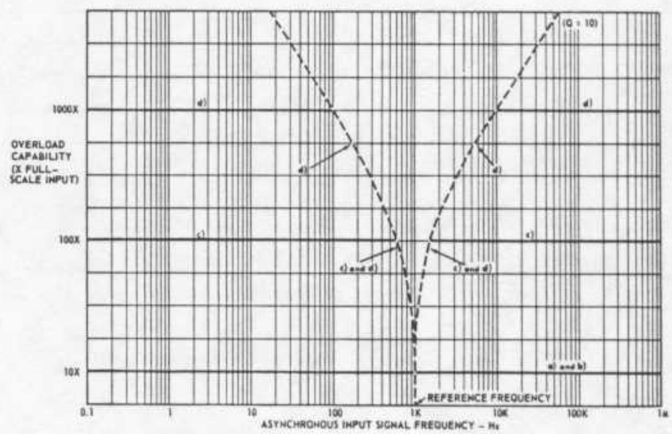

FIGURE F2 EFFECT ON OVERLOAD CAPABILITY OF FRONT–END FILTER POSITION

As already mentioned, interfering signals having a peak amplitude above the overload level will cause gross output errors. It should be noted that large interfering signals having amplitudes *below* the overload level may cause minor output errors due to non-linearities in the signal channel.

It is important that lock-in amplifiers have a high overload capability. Increasing the overload capability ad infinitum is somewhat pointless however, as the following example will show.

Suppose we have a hypothetical lock-in amplifier with an infinite overload capability and use it to measure a very noisy signal. Since the signal/noise improvement caused by the lock-in increases with increasing time-constant (see appendix C), assume that a time-constant value of 125 seconds can be used which corresponds to a measurement time of about 14 minutes and a noise bandwidth b_n of 0.001Hz. Assume that the minimum acceptable value of signal/noise ratio at the output of the lock-in is 10:1. The signal/noise improvement caused by the lock-in is given by $\sqrt{\frac{B_n}{b_n}}$ where B_n is the bandwidth containing the input noise and the larger the value of B_n, the larger the the noise. The largest possible value of B_n will be limited by the input noise bandwidth of the lock-in itself – let B_n = 100kHz.

Then the signal/noise improvement from the input of the lock-in to its output will be given by $\sqrt{\frac{B_n}{b_n}} = \sqrt{\frac{10^5}{10^{-3}}} = 10^4$.

Since the output signal/noise ratio is 10, the input ratio must therefore be $\frac{1}{1000}$. That is the rms value of the input noise is 1000 times larger than the rms value of the input signal. With Gaussian noise, the peak value of the input noise will be greater than 5000 times the rms input signal for less than 0.0001% of the time. For such a measurement then, a lock-in amplifier with an overload capability of $\frac{5000}{\sqrt{2}} \triangleq 3500$, would be perfectly adequate. If the input noise is increased, such a lock-in might overload but even with the hypothetical instrument with infinite overload capability, the output signal/noise ratio would fall below the acceptable minimum value.

In general then, extremely high overload specifications are more impressive than useful except possible for rare cases when the input signal is accompanied by an enormous discrete frequency interfering signal.

APPENDIX G – FRONT–END FILTERING

Many lock-in amplifiers have provisions for narrowing the bandwidth of their front-end ac amplifiers. The reason for such filtering is to improve the overload capability of the instrument as described in appendix F and to attenuate or eliminate the harmonic responses described in appendix D. *Such front-end filtering normally has no effect whatever on the signal to noise improvement effected by the instrument.*

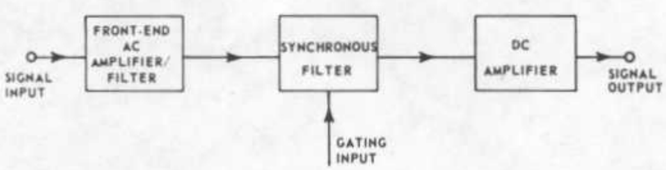

FIGURE G1 SIMPLIFIED SIGNAL CHANNEL

As shown in figure G1, the signal channel of a lock-in amplifier consists of two filters in series, i.e. the front-end filter and the synchronous filter formed by the combined phase-sensitive detector (PSD) and output low-pass filter (LPF) circuits. (see appendix D)

The Q of any bandpass filter is defined (see figure G2)as

$$Q = \frac{\text{center-frequency } (f_o)}{-3\text{dB bandwidth } (2f_c)}$$

For the synchronous filter, the –3dB bandwidth is related to the output time-constant (RC) setting by

$$f_c = \frac{1}{9.76RC} \text{ (for 12dB/oct roll-off – see appendix C)}$$

so that the Q of the synchronous filter is given by

$$Q = 9.76RCf_o$$

If RC = 1 second and f_o = 1kHz for example, the Q of the synchronous filter will be

$$Q = 9.76 \cdot 10^3$$

so that even a Q of 100 in the front-end filter will have no effect.

For maximum overload capability, the optimum position in the signal channel for a filter is at the signal input before any amplification. In commercial lock-in amplifiers however, any filtering is normally placed *after* the front-end amplifier, or at best, with only a small amount of gain between the filter output and the PSD input. The reason for this is that the self-noise of a lock-in amplifier is one of its most important characteristics and filters are inherently noisy.

For this reason, the improvement in overload capability caused by front-end filtering is usually very much less than that indicated by the gain versus frequency response of the filter.

There are three basic types of lock-in amplifiers available and these can be categorized as follows by the type of front-end filtering provided

1. Broadband – This type of instrument uses a broadband (or wide-band) ac amplifier in its front-end, which has amplitude and phase characteristics similar to those shown in figure G3 (curve A).

   The advantage of this type of instrument is that, since its gain and phase-shift are constant over a wide frequency range, it can be used to measure signals of changing frequency. It is also simple to use since there are no frequency controls, though some instruments of this type have some filtering capability such as a notch filter at the power line frequency or simple –6dB/octave low-pass and high-pass filters. Curve B in figure G3 shows the effect on the gain and phase response of such an amplifier/filter when the –3dB frequencies of such low-pass and high-pass filters are set to 300Hz and 30Hz respectively so as to form a bandpass response around a reference frequency of 100Hz. It is interesting to note that even with this simple filtering, a

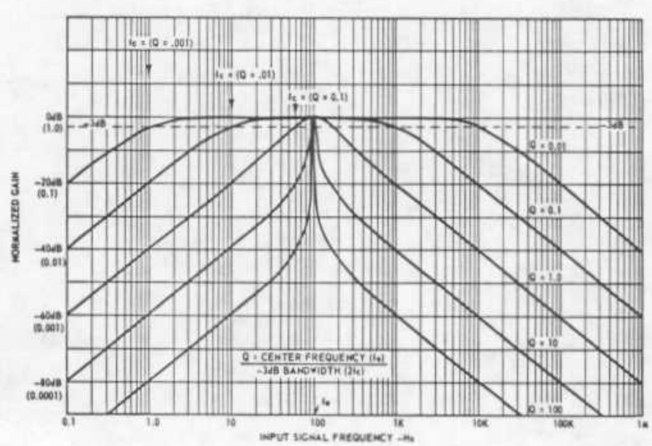

FIGURE G2 Q CURVES

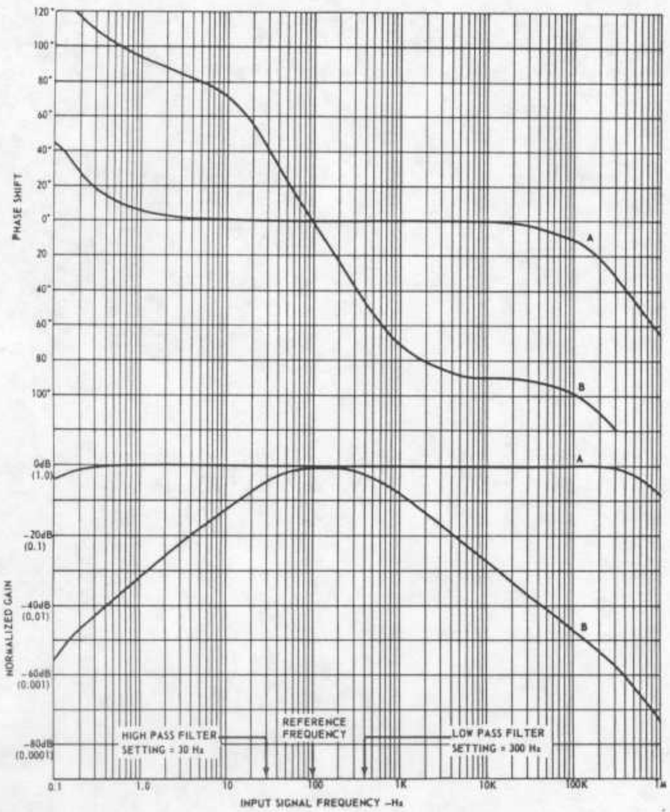

FIGURE G3 EFFECT OF LOW–PASS/HIGH–PASS FILTERING

2:1 change in reference frequency will cause a phase shift of about 24° and hence an output error due to phase-shift alone of almost 9%.

The disadvantage of the wide-band instrument (curve A) is that it will amplify signal and noise equally, so that its overload capability (see appendix F) is that of its phase-sensitive detector alone. The wide-band amplifier plus low-pass/high-pass filtering approach (curve B) provides effective attenuation of interfering signals at frequencies far removed from the reference frequency but provides little attenuation to interference or noise close to the reference frequency.

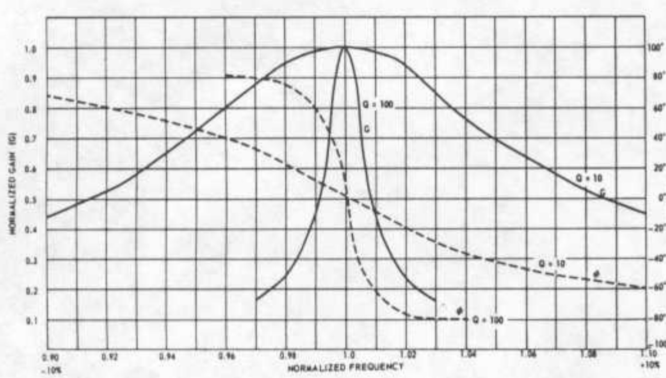

FIGURE G4 GAIN AND PHASE RESPONSE WITH Q = 10 AND Q = 100

uned – A popular means of providing a front-end ilter in a lock-in amplifier is to use a variable frequency, bandpass filter the Q of which can be varied (see figure G4).

A typical range of Q is 1–100 and figure G4 shows the gain and phase response with frequency for Q values of 10 and 100 (on linear scale). The advantage of this approach is that it provides the greatest attenuation of interfering signals close to the reference frequency.

The obvious disadvantage of this approach is that with a Q of 100 for example, a change in signal frequency of as little as 0.1% will cause a phase-shift of more than 11° and an output error of about 2%. A second disadvantage is that with high values of Q such as Q = 100, instability of the filter center frequency f_o with time and temperature will cause significant output errors even if the input frequency is constant.

With low-frequency signals, it is possible to have a Q in the front-end filter that is of the same order or even greater than that of the synchronous filter.

For example, with $f_r = f_o = 6.36$Hz and a time constant setting of 1.25 seconds, the Q of the synchronous filter will be

$$Q = 9.78RCf_o = 9.78 \times 1.25 \times 6.36 = \underline{77.7}$$

More importantly the effective noise bandwidth (see appendices C and D) b_n of the synchronous filter will be

$$b_n = \frac{1}{8RC} = \frac{1}{8 \cdot 1.25} = \underline{0.1\text{Hz}}$$

and for a step change in input signal, the measurement time required for the output of the synchronous filter to rise to within 1% of the final value (see appendix C) will be

$$6.6RC = \underline{8.25 \text{ seconds}}$$

A front-end bandpass filter of Q = 100 will have a time-constant (RC) associated with it given by

$$f_c = \frac{1}{2\pi RC}$$

and since $Q = \dfrac{f_o}{2f_c}$

$$RC = \frac{Q}{\pi f_o} = \frac{100}{6.36\pi} = 5 \text{ seconds}$$

The equivalent noise bandwidth (b_n) of the front-end filter is given by

$$b_n = \frac{\pi}{2} \cdot 2f_c = \frac{1}{2RC} = \frac{1}{2 \cdot 5} = \underline{0.1\text{Hz}}$$

For a step change in input signal, the measurement time required for the output of the front-end filter to rise within 1% of its final value (see appendix C) will be 4.6RC = 23.0 seconds. What this example shows, is that for the same noise bandwidth, the synchronous filter requires a much shorter measurement time than does a front-end bandpass filter. The

Q of a front-end filter should always be at the minimum value required to prevent overload. Any required reduction in noise bandwidth (and therefore output noise) can be effected much more efficiently by increasing the output time-constant of the instrument.

When comparing the shorted-input, self-noise of a tuned front-end lock-in amplifier with a wide-band instrument or with the DYNATRAC 391A, it is important that the additional time-constant of the tuned instrument be taken into account or reduced to zero by reducing the Q setting.

3. Tracking Filter (Heterodyning) – As already described in the main body of this note, the heterodyning front-end used in the DYNATRAC 391A eliminates harmonic responses and provides a bandpass filter characteristic which automatically tracks changes in input signal frequency with no change in phase.

APPENDIX H – REFERENCE CHANNEL TRACKING

It is important to distinguish between signal channel frequency tracking such as that provided by the DYNATRAC 391A and reference channel frequency tracking. The DYNATRAC 391A and almost all lock-ins on the market today have reference circuits which track variations in frequency of the reference signal.

Suppose for example, that with a reference frequency of 1kHz, a phase-shift setting of 30° is required to peak the signal output of an instrument. If the frequency of the reference signal now changes to say 10kHz then the phase-difference between the reference signal and the gating input to the PSD will be maintained constant at 30° (see figure H1).

This frequency tracking ability is a highly desirable feature since it means that the input signal to the PSD will be accurately demodulated regardless of its frequency (within the reference tracking range of the instrument) if the phase-difference between the two PSD inputs is independent of frequency. In fact, in many experiments, the reference and signal inputs to a lock-in *do* have a phase-difference that is constant and independent of frequency. If however, the ac amplifier connected between the signal input to the lock-in and the signal input to the PSD does not provide a constant phase-shift regardless of frequency, then the phase-shift provided by the reference circuits will have to be adjusted for each change in frequency. With experiments where the signal (and reference) frequency is continuously changing (e.g. swept), it is essential that the signal phase-shift caused by the ac amplifier does not change with frequency.

Only the DYNATRAC 391A or wide-band lock-in amplifiers (see appendix G) can measure such swept frequency signals and the front-end tracking filter of the DYNATRAC 391A provides superior overload capability.

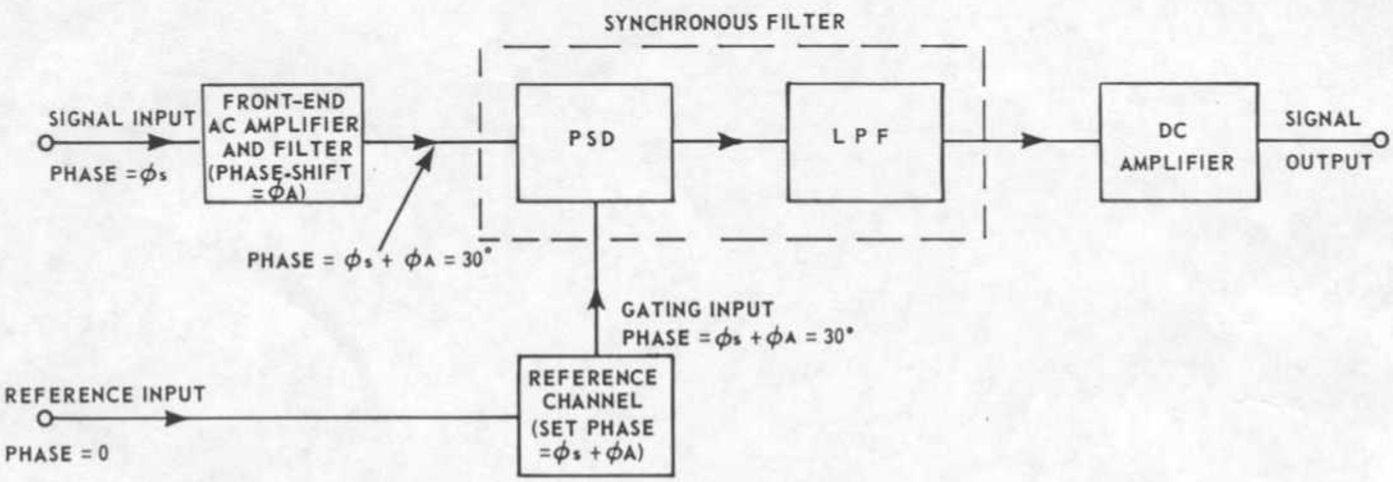

FIGURE H1 PHASE SHIFT EXAMPLE